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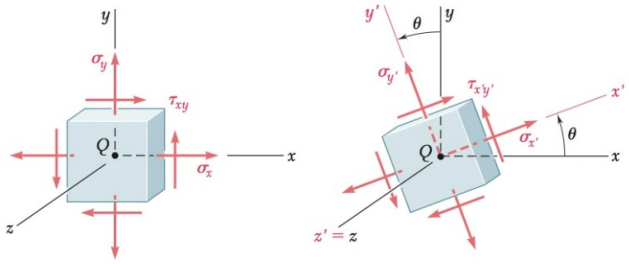
PRINCIPAL STRESSES

Plane Stress at other Angles

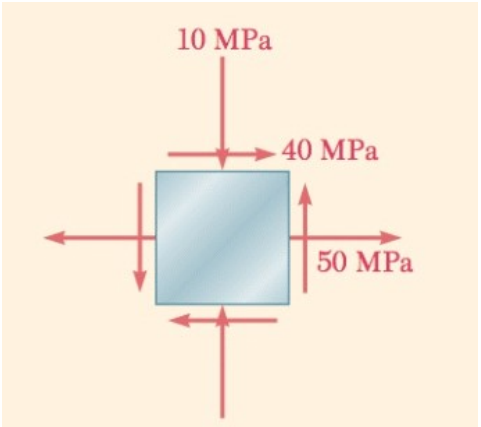


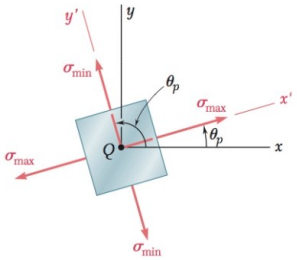
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stresses





$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

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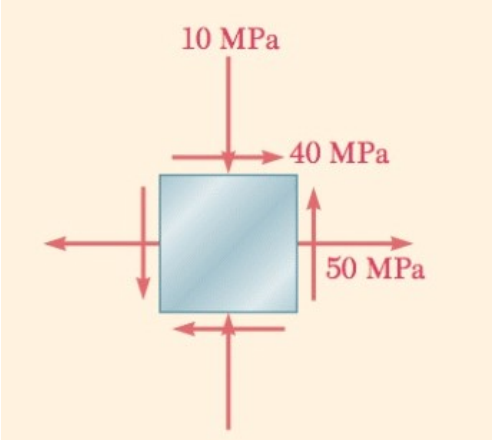
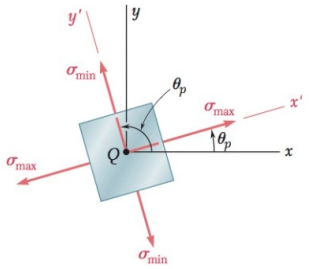
PRINCIPAL STRESSES

Planes





$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(+40)}{50 - (-10)} = \frac{80}{60}$$

$$2\theta_p = 53.1° \quad \text{and} \quad 180° + 53.1° = 233.1°$$

$$\theta_p = 26.6° \quad \text{and} \quad 116.6°$$

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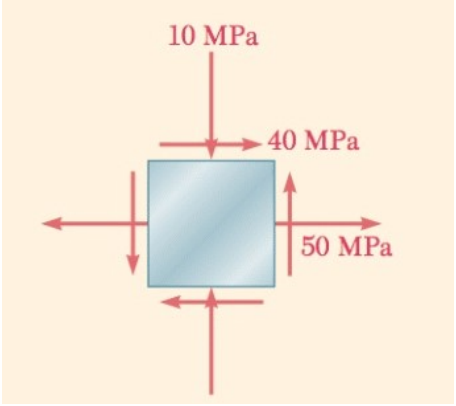
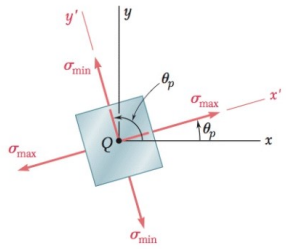
PRINCIPAL STRESSES

Values for Principal Stresses





$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 20 \pm \sqrt{(30)^2 + (40)^2}$$

$$\sigma_{\max} = 20 + 50 = 70 \text{ MPa}$$

$$\sigma_{\min} = 20 - 50 = -30 \text{ MPa}$$

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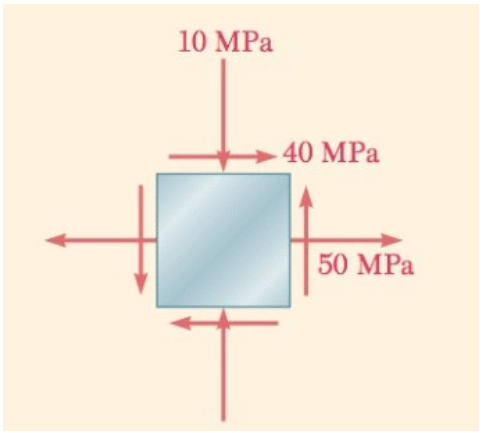
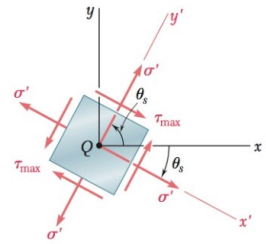
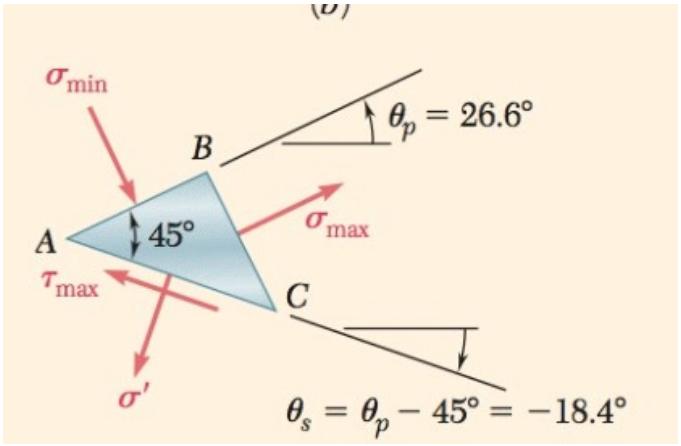
PRINCIPAL STRESSES

Maximum Shear Stress Plane





$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$



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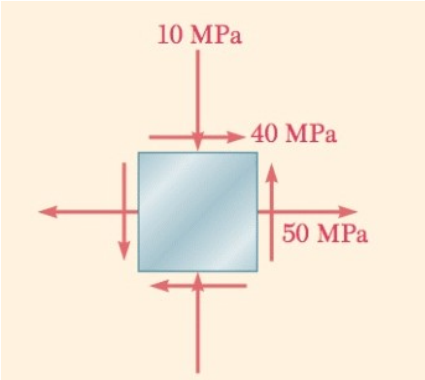
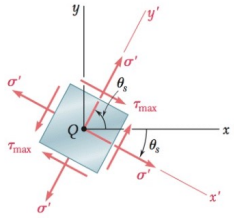
PRINCIPAL STRESSES

Maximum Shear Stress Values





$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(30)^2 + (40)^2} = 50\ \text{MPa}$$

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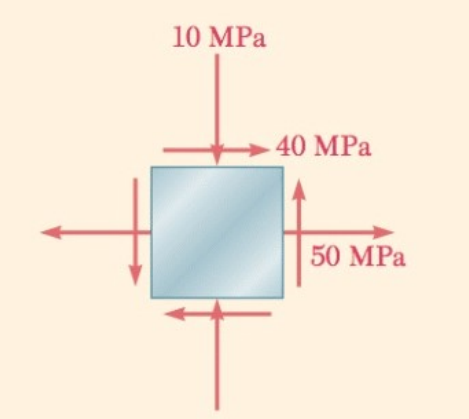
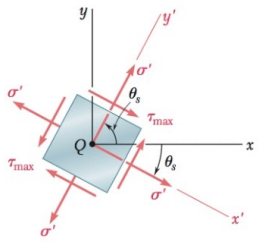
PRINCIPAL STRESSES

Corresponding Normal Stresses





$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 - 10}{2} = 20\ \text{MPa}$$

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PRINCIPAL STRESSES

Sample Problem Elaboration

$$P = 150\text{ lb} \qquad T = (150\text{ lb})(18\text{ in.}) = 2.7\text{ kip·in.}$$

$$M_x = (150\text{ lb})(10\text{ in.}) = 1.5\text{ kip·in.}$$

$$\sigma_x = 0 \quad \sigma_y = +\frac{Mc}{I} = +\frac{(1.5\text{ kip·in.})(0.6\text{ in.})}{\frac{1}{4}\pi\,(0.6\text{ in.})^4} \qquad \sigma_y = +8.84\text{ ksi}$$

$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(2.7\text{ kip·in.})(0.6\text{ in.})}{\frac{1}{2}\pi\,(0.6\text{ in.})^4} \qquad \tau_{xy} = +7.96\text{ ksi}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 8.84}{2} \pm \sqrt{\left(\frac{0 - 8.84}{2}\right)^2 + (7.96)^2} = +4.42 \pm 9.10$$

$$\sigma_{\max} = +13.52\text{ ksi}$$

$$\sigma_{\min} = -4.68\text{ ksi}$$

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PRINCIPAL STRESSES

Sample Problem Elaboration

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(7.96)}{0 - 8.84} = -1.80$$

$$2\theta_p = -61.0^\circ \quad \text{and} \quad 180^\circ - 61.0^\circ = +119^\circ$$

$$\theta_p = -30.5^\circ \quad \text{and} \quad +59.5^\circ \blacktriangleleft$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$